NATUURKUNDE OLYMPIADE 2013 Eindronde
ANTWOORDEN Theoretische toets

Opgave 1 Helikopter

The required power for the hovering helicopter depends on the gravitational acceleration g, the linear size of the helicopter L, the average density of the helicopter ρ_{hel}, and the density of air ρ_{air}.
It is reasonable to assume that the mechanical power needed depends only on these quantities and that the dependence is a power relationship:

$$P \propto g^{\alpha} \times L^{\beta} \times \rho_{hel}^{\gamma} \times \rho_{air}^{\delta}$$

The dimensions of the left- and right-hand sides must be equal:

$$\frac{\text{kg m}^2}{\text{s}^3} = \left(\frac{\text{m}}{\text{s}^2}\right)^{\alpha} \times \text{m}^{\beta} \times \left(\frac{\text{kg}}{\text{m}^3}\right)^{\gamma} \times \left(\frac{\text{kg}}{\text{m}^3}\right)^{\delta}$$

which yields

$$\gamma + \delta = 1$$
$$\alpha + \beta - 3(\gamma + \delta) = 2$$
$$-2\alpha = -3$$

The solution of this system of linear equations is $\beta = \frac{7}{2}$, $\alpha = \frac{3}{2}$ and $\gamma = 1 - \delta$.

It can be seen that the mechanical power needed is proportional to the $\frac{7}{2}$ power of the linear size. Consequently, the second helicopter should have an engine producing power

$$\left(\frac{1}{2}\right)^{7/2} P = 0,088P$$

Note.

(i) The efficiency of a mechanical engine can be characterised by the ratio of the power produced to the mass of the engine. According to the above result the 'specific power'

$$\frac{P}{m} \propto \frac{P}{L^3} \propto \sqrt{L}$$

i.e. the efficiency required increases as the linear size increases. This means that the smaller a helicopter is, the more easily it can hover. There are many small animals (bees, dragonflies, hummingbirds, etc.) that can hover like a helicopter, but larger birds are unable to do so.

(ii) Using simple dimensional analysis we could find only the *sum* of the exponents γ and δ However, it is clear that P can depend only on the product of the density of the helicopter and g, because, when the helicopter is hovering, the relevant quantity is not its inertial mass, but its weight.
Thus γ must be equal to α, i.e. $\gamma = \frac{3}{2}$ with $\delta = -\frac{1}{2}$. Finally, we get

$$P \propto \left(g\,\rho_{hel}\right)^{3/2} \times L^{7/2} \times \rho_{air}^{-\frac{1}{2}} = \left(L^3 \rho_{hel} g\right) \times \sqrt{Lg} \times \sqrt{\frac{\rho_{hel}}{\rho_{air}}}$$

Here, on the surface of the Earth, we can change only the size and density of the helicopter. Nevertheless, for a space mission using robot helicopters, it could be useful to know how P depends on the gravitational acceleration and the atmospheric density of the target planet.

Opgave 2 Looping

Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.

$$\sum F_{\text{bottom}} = F_{\underset{\text{bottom}}{\text{N}}} - mg = m v_{\text{bottom}}^2 / R \quad \rightarrow \quad F_{\underset{\text{bottom}}{\text{N}}} = mg + m v_{\text{bottom}}^2 / R$$

Now consider the force diagram at the top of the loop. Again, the net force must be centripetal, and so must be downward.

$$\sum F_{\text{top}} = F_{\underset{\text{top}}{\text{N}}} + mg = m v_{\text{top}}^2 / R \quad \rightarrow \quad F_{\underset{\text{top}}{N}} = m v_{\text{top}}^2 / R - mg \, .$$

Assume that the speed at the top is large enough that

$F_{\underset{\text{top}}{N}} > 0$ and so $v_{\text{top}} > \sqrt{Rg}$

Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for potential energy $\left(y = 0\right)$.

We have $y_1 = 0$ and $y_2 = 2R$.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2} m v_1^2 + mgy_1 = \tfrac{1}{2} m v_2^2 + mgy_2 \quad \rightarrow \quad v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR$$

The difference in apparent weights is the difference in the normal forces.

$$F_{\underset{\text{bottom}}{\text{N}}} - F_{\underset{\text{top}}{\text{N}}} = \left(mg + m v_{\text{bottom}}^2 / R\right) - \left(m v_{\text{top}}^2 / R - mg\right) = 2mg + m\left(v_{\text{bottom}}^2 - v_{\text{top}}^2\right) / R$$

$$= 2mg + m\left(4gR\right)/R = \boxed{6mg}$$

Notice that the result does not depend on either v or R.

Opgave 3 Membraan in zuiger

(a) Gas B will undergo adiabatic compression and produce heat which will increase the temperature of both gases. However, since both gases are in thermal contact, their temperature will be the same. Furthermore, in the case of gas A, its temperature will be the same on both sides of the membrane M.

Since the piston P_1 is allowed to move, the pressure of gas A will be unchanged, i.e.

$\frac{V}{T}$ = constant

Now the volume occupied by gas A is the total volume on both sides of the membrane. Hence, the final temperature of the gases is:

$$T_f = \frac{0,02}{0,01} \cdot 300 = 600 \text{ K}$$

(b) Applying the 1st law of thermodynamics to the whole system:

$$dU_A + dU_B = -p_A dV_A + p_B dV_B$$

Because the two gases are in thermal equilibrium:

$$dU_A = dU_B = c_V dT = \tfrac{3}{2} R dT$$

Hence:

$$3\int_{T_0}^{T_f} \frac{dT}{T} = -\int_{V_0}^{2V_0} \frac{dV_A}{V_A} - \int_{V_0}^{V_B} \frac{dV_B}{V_B}$$

where V_0 is the initial volume and V_B is the final volume for gas B.

So:

$$3\ln 2 = -\ln 2 - \ln\frac{V_B}{V_0}$$

$$4\ln 2 = \ln\frac{V_0}{V_B}$$

$$V_B = \tfrac{1}{16}V_0$$

Hence, the volume occupied by gas A between the membrane M and the piston P_1 is: (0,02 – 0,000625) m^3 = 0,019375 m^3

(c) Since the pressure and temperature of gas A is the same on both sides of the membrane, then the no. of moles of the gas will be proportional to its volume.
So no. of moles in the volume between M en P_1 will be:

$$\frac{0,019375}{0,02} = 0.96875$$

Opgave 4 Neutronen interferometer

The neutron behaves both as an particle as well as a wave. As a particle of finite mass m, the neutron loses K.E. as it travels vertical distance b given by $\Delta E = mgb$.

As a wave, the neutron has a De Broglie wavelength of $\lambda = h/p$ where $p = \sqrt{2mE}$ is the momentum.

Hence, over a distance a, the number of oscillations made is a/λ.

For a certain setting of b, the two neutron beams will make the same number of oscillations along the vertical sites of the rectangle.

Their difference in number of oscillations occurs only along the horizontal paths DC and AB and is given by:

$$n' = \frac{a}{\lambda} - \frac{a}{\lambda'} = \frac{a}{h}\left(\sqrt{2mE} - \sqrt{2m(E-\Delta E)}\right)$$

$$= \frac{a\sqrt{2mE}}{h}\left(1 - \sqrt{1 - \frac{\Delta E}{E}}\right)$$

Met een taylorreeksontwikkeling (ΔE is klein t.o.v. E), levert dat:

$$n' = \frac{a\sqrt{m}}{h\sqrt{2E}}mgb$$

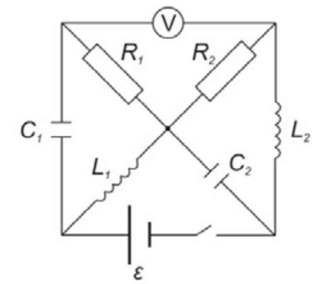
The number of *amplitude* oscillations when b changes by a distance x is

$$n'' = \frac{a\sqrt{m}}{h\sqrt{2E}}mgx$$

The number of intensity oscillations over this distance is:

$$n = 2n'' = \frac{2a\sqrt{m}}{h\sqrt{2E}}mgx = \frac{\sqrt{2}m^{3/2}a}{h\sqrt{E}}gx$$

Opgave 5 Schakeling



(a) In the stationary regime, the capacitors can be effectively disconnected (they conduct no direct current) and the inductors can be substituted by wires. If the voltmeter is ideal, there is therefore no current through R_1 and the voltmeter shows the voltage on R_2 equalling ε.

(b) Capacitors cannot immediately change their voltage and inductors cannot instantaneously change their current. L_1 and L_2 had both been carrying all the current that had been flowing through the circuit, hence, after opening the switch, they still carry a current of ε/R_2 and act as such current sources. As the current from L_2 flows also through R_2, the voltage on R_2 is ε (with the "+"-side at the centre of the circuit). The current through L_1 flows also through R_1 (it is the current charging C_1); therefore the voltage on R_1 is $\varepsilon\, R_1/R_2$ (with the "+"-side at the centre). The reading of the voltmeter has changed its sign and is $\varepsilon\,(1 - R_1/R_2)$ or, plugging in the data, -2ε .

(c) Immediately after opening the switch, capacitor C_1 was uncharged (it had been parallel to R_1 that was carrying no current) and C_2 had a voltage of ε (it had been directly parallel to the battery). L_1 and L_2 were both carrying a current of ε/R_2. The voltmeter can be effectively disconnected (its resistance is huge), giving us two separate circuits, $R_1\, L_1\, C_1$ and $R_2\, L_2\, C_2$.

Therefore (by the potential energy formulae $CU^2/2$ and $LI^2/2$) the energy stored in the left-hand circuit was $L_1\varepsilon^2/2R_2^2$ or, with the given data, $L\varepsilon^2/(2R^2)$. This is the energy dissipated from R_1.

The corresponding expressions for the right-hand circuit (giving the energy dissipated from R_2) are $C_2\varepsilon^2/2 + L_2\varepsilon^2/2R_2^2$ and $C\varepsilon^2/2 + L\varepsilon^2/2R^2$.

Opgave 6 Tillen door stroom

(a) The Ampère force pulls the wires to the side so that the wires will take a curved shape. Since the Ampère force is perpendicular to the wire, the mechanical tension is constant along the wires.
Let the tension be T and the curvature of the wire at a certain point R. Let us consider a short piece of the wire, of length $a \ll R$. Then the angle by which the tangent of the wire rotates while the tangent point moves over an arc of length a is given by $\alpha = a/R$. Let us study the force balance in the perpendicular direction for that piece of wire: the Ampère's force IaB is balanced by the tension $T\alpha = T\,a/R$.
So, $R = T/IB$ which means that R is constant, and the wire will take the form of a circle segment. To conclude, both halves of the wire will take the form of a circle segment, the convex sides of which are turned outside.

(b) The maximal height is achieved when the circle segments form a perfect circle, in which case the lifting height is

$$\Delta h = l - \frac{2l}{\pi} = l\left(1 - \frac{2}{\pi}\right)$$

(c) If the central angle of the circle segments is 2α, the tangents to the wires at the point where the load is fixed forms angle α with the vertical direction. So, the lifting force is $mg = 2T\cos\alpha$. From the other hand, $R = l/2\alpha = T/IB$ ie.

$$\alpha\frac{mg}{lIB} = \cos\alpha$$

which is the equation from where one can determine the angle α. Then, the lifting height:

$$\Delta h = l - 2R\sin\alpha = l - \frac{2T}{lIB}\sin\alpha = l\left(1 - \frac{\sin\alpha}{\alpha}\right)$$

(d) From the previous result it can be seen that we need to have:

$\dfrac{\sin\alpha}{\alpha} = \dfrac{\pi}{3}$, hence $\alpha = \dfrac{\pi}{6}$ and

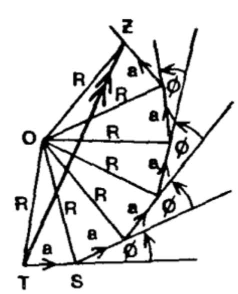
$$I = \frac{mg\alpha}{lB\cos\alpha} = \frac{mg\pi}{3\sqrt{3}lB}$$

Opgave 7 Lichtgolf

(a) Neem aan dat de fase van de eerste spleet 0 is. Dan geldt voor de fase van de tweede spleet: $\phi = \frac{2\pi}{\lambda} d\sin\theta$.
Neem voor het gemak even $\epsilon = 2\pi\left(ft - \frac{x}{\lambda}\right)$,
Dan kun je voor de som van golven schrijven:

$$g_1 + g_2 = a\cos(\epsilon + \phi) + a\cos(\epsilon) = 2a\cos\left(\frac{\phi}{2}\right)\cos\left(\epsilon + \frac{\phi}{2}\right) = 2a\cos(\beta)\cos(\epsilon + \beta)$$

met $\beta = \frac{\phi}{2} = \frac{\pi}{\lambda}\sin\theta$.
Hieruit volgt dat de amplitude moet zijn $A = 2a\cos\beta$



(b) Elke spleet geeft een golf met amplitude a met een fase 2β ten opzichte van de vorige spleet. Je kunt deze in een

fasordiagram uitzetten. Met N spleten krijg je een regelmatige veelhoek met constante zijden a en een constante hoek tussen opvolgende zijden.
$O$is het middelpunt van de omschreven cirkel door de uiteinden van de fasoren. Zijden als OS hebben dan lengte R.
$\angle OST = \angle OTS = \frac{1}{2}(\pi - \phi)$ en $\angle TOS = \phi$
In driehoek TOS geldt dan:
$a = 2R\sin\left(\frac{\phi}{2}\right) = 2R\sin\beta$ met $\phi = 2\beta$
$R = \dfrac{a}{2\sin\beta}$
Met N driehoeken in de veelhoek geldt: $\angle TOZ = N\angle TOS = N\varphi = 2N\beta$
Daarom kun je de totale amplitude Z van de resulterende golf noteren vanuit driehoek TOZ als $Z = 2R\sin N\beta$
Met wat we al voor R vonden levert dat: $Z = \frac{a\sin N\beta}{\sin\beta}$

Opgave 8 Bowlen

De bal zal eerst glijden maar door de wrijvingskracht zal er ook rotatie ontstaat. Na een zekere tijd zal de bal overgaan in rollen.
Voor de translatie snelheid van de bal geldt:

$$v = u - \frac{F}{m}t = u - \mu gt$$

waarbij gebruikt is dat $F = \mu mg$. Voor de rotatie geldt $FR = I\alpha$, en samen met $I = \frac{2}{5}MR^2$ volgt voor de hoekversnelling:

$$\alpha = \frac{5}{2}\frac{\mu g}{R}$$

De rotatiesnelheid van de bal is dus:

$$v_R = R\alpha t = \tfrac{5}{2}\mu gt$$

Op het moment dat het glijden overgaat op rollen, geldt $v = v_R$, oftewel:

$$u - \mu gt = \tfrac{5}{2}\mu gt$$

Hieruit is het tijdstip te berekenen van de overgang van glijden naar rollen.

$$t = \tfrac{2}{7}\frac{u}{\mu g} = 2{,}04 \text{ s}$$

De translatiesnelheid van de bal op dat moment is:

$$v = u - \mu gt = 7{,}0 - 0{,}10 \cdot 9{,}8 \cdot 2{,}04 = 5{,}0 \text{ m/s}$$

De beweging is eenparig vertraagd, dus de gemiddelde snelheid in het glijgedeelte is 6,0 m/s. De afstand die tijdens het glijden wordt afgelegd is dus 2,04 · 6,0 = 12,2 m.
De bal moet nog over een afstand van 18,5 – 12,2 = 6,3 m rollen met een snelheid van 5,0 m/s. Daar doet de bal 1,26 s over.
De totale tijd wordt dus: 2,04 + 1,26 = 3,3 s.

Opgave 9 Luchtballon (NTvN, mei 2013)

De straal R van de ballon is 10 m. Het volume V van de ballon bedraagt:

$$V = \tfrac{4}{3}\pi R^3 = 4,19\cdot 10^3 \text{ m}^3$$

Het aantal mol lucht in de ballon bij 280 K is dus:

$$n = \frac{pV}{RT} = \frac{1,0\cdot 10^5 \cdot 4,19\cdot 10^3}{8,31\cdot 280} = 1,80\cdot 10^5 \text{ mol}$$

De luchtmassa is dus:

$$0,029\cdot 1,80\cdot 10^5 = 5,22\cdot 10^3 \text{ kg}$$

Als de massa 420kg lager wordt kan de ballon opstijgen. De luchtmassa in de ballon mag dus maximaal $4,80 \cdot 10^3$ kg zijn.

Dit komt overeen met:

$$\frac{4,80\cdot 10^3}{0,029} = 1,66\cdot 10^5 \text{ mol}$$

lucht.

De minimale temperatuur van de lucht in de ballon moet dus zijn:

$$T = \frac{pV}{nR} = \frac{1,0\cdot 10^5 \cdot 4,19\cdot 10^3}{1,66\cdot 10^5 \cdot 8,31} = 304,5 \text{ K}$$

Opgave 10 Ruimteschepen

(a) Impliciete aanname is gelijktijdigheid. De Amerikaans gezagvoerder laat de bom op hetzelfde moment ontploffen, maar dit is natuurlijk niet gelijktijdig voor de Chinezen. De Amerikanen hebben dus gelijk: de bom zal te vroeg ontploffen om het Chinese schip te kunnen raken.

(b) De coördinaten zijn (x, ct) = (100,0) voor de Amerikanen en (125, -75) voor de Chinezen, alles in meter. De laatste geeft aan dat de bom zal ontploffen voordat de Amerikanen langszij komen.