Woensdag 22 april 2015

1 Eenheden. (Introduction to classical Mechanics, Morin. Problem 1.3)

We want to make a speed, $[v] = L/T$, out of the quantities $[\rho] = M/L^3$, and $[B] = [F/A] = (ML/T^2)/L^2 = M/(LT^2)$. We can play around with these quantities to find the combination that has the correct units, but let's do it the no-fail way. If $v \propto \rho^a B^b$, then:

$$\frac{L}{T} = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{LT^2}\right)^b$$

Matching up the powers of the three kinds of units on each side of this equation gives:

$$M: 0 = a + b$$
$$L: 1 = -3a - b$$
$$T: -1 = -2b$$

The solution to this system of equations is $a = -\frac{1}{2}$ and $b = \frac{1}{2}$.

Therefore, our answer is $v \propto \sqrt{B/\rho}$

Fortunately, there was a solution to this system of three equations in two unknowns.

2 Schuin stuiteren. (Engineering Mechanics, Hibbeler, Problem 15-75)

(a) Conservation of Energy.

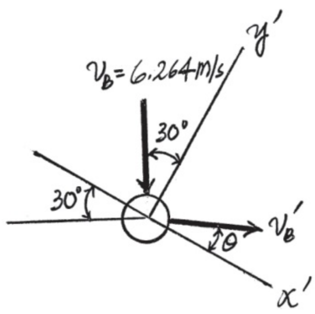
$T_A + V_A = T_B + V_B$

$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$

$0 + 1 \cdot 9{,}81 \cdot 2 = \frac{1}{2} 1 \cdot v_B^2 + 0$

$v_B = 6{,}264 \text{ m/s} \downarrow$

(b) First in the x' direction. Conservation of Linear Momentum. Since no impulsive force acts on the ball along the inclined plane (x' axis) during impact, linear momentum of the ball in conserved along the x' axis. Referring to the figure.



$m(v_B)_{x'} = m(v'_B)_{x'}$

$1 \cdot 6{,}264 \cdot \sin 30° = 1 \cdot v'_B \cos\theta$

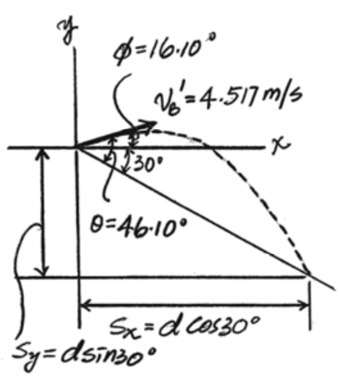
$v'_B \cos\theta = 3{,}1321$

Second in the y' direction. Coefficient of Restitution. Since the inclined plane does not move during impact,

$e = \frac{(v'_B)_{y'}}{(v_B)_{y'}} = \frac{v'_B \sin\theta}{v_B \cos 30°} = 0{,}6$ so $v'_B \sin\theta = 3{,}2550$

Solving both equations yields $\theta = 46{,}10°$ and $v'_B = 4{,}517 \text{ m/s}$

(c) Kinematics. By considering the x and y motion of the ball after impact, see figure.



$s_x = (s_0)_x + (v'_B)_x t$

$d\cos 30° = 0 + 4{,}517\cos(16{,}10°)t$

$t = 0{,}1995d$

$s_y = (s_0)_y + (v'_B)_y t + \frac{1}{2}a_y t^2$

$-d\sin 30° = 0 + 4{,}517\sin(16{,}10°)\,t + \frac{1}{2}(-9{,}81)t^2$

$4{,}905t^2 + 1{,}252t - 0{,}5d = 0$

Solving the two equations yields:

$d = 3{,}84 \text{ m}$ (and $t = 0{,}7663 \text{ s}$)

3 **Trillend staafje.** (SNON Eindronde 2001)

(a) De totale weerstand van de draden is: $R = \rho \frac{2L}{\frac{\pi}{4}D^2} = 0{,}14\ \Omega$

Dus geldt voor de stroom: $I = \frac{V}{R} = \frac{10}{0{,}14} = 71\ \mathrm{A}$.

(b) Op kleine afstand x van een lange draad is het magnetisch veld: $B = \frac{\mu_0}{2\pi}\frac{I}{x}$

In het midden van beide draden is het veld dus: $B = 2 \cdot 2{,}0 \cdot 10^{-7}\frac{71}{0{,}05} = 5{,}7 \cdot 10^{-4}\ \mathrm{T}$

(c) Stel het staafje verschuift over een kleine afstand x, dan hebben de draden in de ene helft een lengte $\frac{1}{2}L - x$, daar loopt een stroom I_1 en in de andere helft hebben de draden een lengte $\frac{1}{2}L + x$ en daar loopt een stroom I_2. De totale stroom door het staafje is dan:

$$I = I_1 - I_2 = \frac{5}{\frac{\rho}{\frac{\pi}{4}D^2}2\left(\frac{1}{2}L - x\right)} - \frac{5}{\frac{\rho}{\frac{\pi}{4}D^2}2\left(\frac{1}{2}L + x\right)} = \frac{5\pi D^2}{\rho}\frac{x}{L^2 - 4x^2} \approx \frac{5\pi D^2}{\rho}\frac{x}{L^2}$$

Dus $I = 143x$

(d) Op het staafje wordt een Lorentzkracht uitgeoefend: $F = BId = 143Bdx$

Uit $F = -m\ddot{x}$ volgt een periode: $T = 2\pi\sqrt{\frac{m}{143Bd}} = 16\ \mathrm{s}$

4 **IJsberg.** (200 Puzzling Physics Problems, P8)

If the berg has base area A and height H, then $M = \frac{1}{3}AH\rho_{ice}$. If the height showing above the surface is h, the floating condition gives $(H^3 - h^3)\rho_{water} = H^3\rho_{ice}$.

When the berg is depressed by a small amount x the additional submerged volume is $xA(h/H)^2$ and the upthrust is this multiplied by $\rho_{water}g$.

This gives the angular frequency of oscillation ω is determined by

$$\omega^2 = \frac{3h^2\rho_{water}g}{\rho_{ice}H^3}$$

And, on substituting numerical values, that the period of oscillation is about 11 s.

5 **Kringproces.** (Giancoli 20.72)

We have a monatomic gas, so $\gamma = \frac{5}{3}$. Also the pressure, volume, and temperature for state a are known. We use the ideal gas law, the adiabatic relationship, and the first law of thermodynamics.

(a) Use the ideal gas equation to relate states a and b. Use the adiabatic relationship to relate states a and c.

$$\frac{P_b V_b}{T_b} = \frac{P_a V_a}{T_a} \quad \rightarrow$$

$$P_b = P_a \frac{V_a}{V_b}\frac{T_b}{T_a} = (1.00\,\mathrm{atm})\left(\frac{22.4\,\mathrm{L}}{56.0\,\mathrm{L}}\right)\left(\frac{273\,\mathrm{K}}{273\,\mathrm{K}}\right) = \boxed{0.400\,\mathrm{atm}}$$

$$P_a V_a^\gamma = P_c V_c^\gamma \quad \rightarrow$$

$$P_c = P_a\left(\frac{V_a}{V_c}\right)^\gamma = (1.00\,\text{atm})\left(\frac{22.4\,\text{L}}{56.0\,\text{L}}\right)^{5/3} = 0.2172\,\text{atm} \approx \boxed{0.217\,\text{atm}}$$

(b) Use the ideal gas equation to calculate the temperature at c.

$$\frac{P_b V_b}{T_b} = \frac{P_c V_c}{T_c} = \quad \rightarrow \quad T_c = T_b \frac{P_c}{P_b}\frac{V_c}{V_b} = (273\,\text{K})\left(\frac{0.2172\,\text{atm}}{0.400\,\text{atm}}\right)(1) = \boxed{148\,\text{K}}$$

(c) Process ab: $\Delta E_{\text{int}_{ab}} = nC_V \Delta T = \boxed{0}$;

$$Q_{ab} = W_{ab} = nRT \ln\frac{V_b}{V_a} = (1.00\,\text{mol})(8.314\,\text{J/mol}\bullet\text{K})(273\,\text{K})\ln 2.5$$

$$= 2079.7\,\text{J} \approx \boxed{2080\,\text{J}}$$

Process bc: $W_{bc} = \boxed{0}$;

$$\Delta E_{\text{int}_{bc}} = Q_{bc} = nC_V \Delta T = (1.00\,\text{mol})\tfrac{3}{2}(8.314\,\text{J/mol}\bullet\text{K})(148\,\text{K} - 273\,\text{K})$$

$$= -1559\,\text{J} \approx \boxed{-1560\,\text{J}}$$

Process ca: $Q_{ca} = \boxed{0}$ (adiabatic) ;

$$\Delta E_{\text{int}_{ca}} = -W = -\Delta E_{\text{int}_{ab}} - \Delta E_{\text{int}_{bc}} = -0 - (-1560\,\text{J}) \quad \rightarrow$$

$$\Delta E_{\text{int}_{ca}} = \boxed{1560\,\text{J}} \ ; \ W_{ca} = \boxed{-1560\,\text{J}}$$

(d) $e = \dfrac{W}{Q_{\text{input}}} = \dfrac{2080\,\text{J} - 1560\,\text{J}}{2080\,\text{J}} = \boxed{0.25}$

6 Vitrage.

Het eerste onderdeel wordt gebruikt om de golflengte van de laserpointer te bepalen.

Het gegeven 500 /mm levert dat $d = \frac{10^{-3}}{500} = 2{,}00 \cdot 10^{-6}$ m.

De eerste orde zit op 76,1 cm. Oftewel: $\tan\alpha = 76{,}1/276$ Dus: $\alpha = 15{,}4°$

Met gebruik van $d\sin\alpha = k\lambda$ is λ te bepalen. In dit geval $k = 1$.

$\lambda = d\sin\alpha = 2{,}00 \cdot 10^{-6} \cdot 0{,}2658 = 532$ nm.

Het tweede onderdeel wordt gebruikt om de afstand tussen de draden van de vitrage te bepalen. Aan de foto te zien, is die afstand in beide richtingen gelijk, immers de eerste ordes liggen in beide richtingen op gelijke afstanden.

De afstand in horizontale richting tussen de eerste ordes is 1,4 cm.

Voor dergelijk kleine hoeken geldt dat $\sin\alpha = \tan\alpha$

Hieruit volgt:

$$d = \frac{\lambda}{\sin\alpha} = \frac{532 \cdot 10^{-9}}{\tan\alpha} = 532 \cdot 10^{-9} \cdot \frac{920}{1{,}4} = 3{,}5 \cdot 10^{-4}\ \text{m}$$

Het aantal draden in zowel horizontale als verticale richting per meter is dus

$\frac{1}{3{,}5} \cdot 10^{-4} = 2860$ dat is dus in totaal 5720 m (5,7 km) per vierkante meter vitrage.

7 Reuzenrad. (SNON 1e ronde 1997)

De gemiddelde normaalkracht is gelijk aan de zwaartekracht: $mg = 455\ N$

De maximale afwijking is gelijk aan de middelpuntzoekende kracht: $\frac{mv^2}{r} = 25N$

Gecombineerd levert dit: $\frac{v^2}{r} = \frac{25}{455}g$

Verder geldt: $v = \frac{2\pi r}{T}$

Zodat: $\frac{v^2}{\left(\frac{Tv}{2\pi}\right)} = \frac{25}{455}g$ oftewel: $v = \frac{25Tg}{2\pi 455} = 3{,}1\ m/s$

8 Pendulum. (Engineering Mechanics, Hibbeler, Problem 19-47)

Traagheidsmoment: $I_A = \frac{1}{3}(4)(2)^2 + \frac{2}{5}(10)(0{,}3)^2 + 10(2{,}3)^2 = 58{,}6\ \mathrm{kg/m^2}$

Vlak voor botsing: $T_1 + V_1 = T_2 + V_2$ dat invullen: $0 + 0 = \frac{1}{2}(58{,}6)\omega^2 - 4\cdot 1\cdot 9{,}81 - 10\cdot 2{,}3\cdot 9{,}81$

$\omega = 3{,}00\ \mathrm{rad/s}$ oftewel: $v = \omega r = 6{,}9\ \mathrm{m/s}$

9 Groter of kleiner? (Giancoli 29.78)

The emf around the loop is equal to the time derivative of the flux. Since the area of the coil is constant, the time derivative of the flux is equal to the derivative of the magnetic field multiplied by the area of the loop. To calculate the emf in the loop we add the voltage drop across the capacitor to the voltage drop across the resistor. The current in the loop is the derivative of the charge on the capacitor.

$$I = \frac{dQ}{dt} = \frac{dCV}{dt} = \frac{d}{dt}\left[CV_0\left(1 - e^{-t/\tau}\right)\right] = \frac{CV_0}{\tau}e^{-t/\tau} = \frac{V_0}{R}e^{-t/\tau}$$

$$\mathcal{E} = IR + V_C = \left(\frac{V_0}{R}e^{-t/\tau}\right)R + V_0\left(1 - e^{-t/\tau}\right) = V_0 = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt} \rightarrow \boxed{\frac{dB}{dt} = \frac{V_0}{\pi r^2}}$$

Since the charge is building up on the top plate of the capacitor, the induced current is flowing clockwise. By Lenz's law this produces a downward flux, so the external downward magnetic field must be decreasing.

10 Virtuele spleet. (Physics, H18 opgave 6)

Afstand tussen twee opvolgende donkere lijnen is 0,90 mm, dus $\frac{\lambda}{d} = \frac{x}{L}$

$x = \frac{\lambda}{d}L = \frac{540\cdot 10^{-9}}{0{,}9\cdot 10^{-3}}0{,}60 = 0{,}36\ \mathrm{mm}.$

Dat is de afstand tussen de twee 'spleten', dus $h = 0{,}18\ \mathrm{mm}.$

11 Michelson interferometer. (Giancoli 34.40 + 34.42)

(a) The path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of λ, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let N be the number of fringe shifts produced by a mirror movement of Δx.

$$N=\frac{\Delta x}{\frac{1}{2}\lambda} \quad \rightarrow \quad \lambda=\frac{2\Delta x}{N}=\frac{2\left(1.25\times10^{-4}\,\mathrm{m}\right)}{384}=6.51\times10^{-7}\,\mathrm{m}=\boxed{651\,\mathrm{nm}}$$

(b) One fringe shift corresponds to an effective change in path length of λ. The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a length *d*, the number of wavelengths in vacuum is $\frac{d}{\lambda}$, and the (greater) number with the gas present is $\frac{d}{\lambda_{\text{gas}}}=\frac{n_{\text{gas}}d}{\lambda}$. Because the light passes through the cavity twice, the number of fringe shifts is twice the difference in the number of wavelengths in the two media.

$$N=2\left(\frac{n_{\text{gas}}d}{\lambda}-\frac{d}{\lambda}\right)=2\frac{d}{\lambda}\left(n_{\text{gas}}-1\right) \quad \rightarrow \quad n_{\text{gas}}=\frac{N\lambda}{2d}+1=\frac{(176)\left(632.8\times10^{-9}\,\mathrm{m}\right)}{2\left(1.155\times10^{-2}\,\mathrm{m}\right)}+1=\boxed{1.00482}$$

12 Even fel. (SNON Eindronde 1999)

Stel de spanning van de batterij V_0 en de weerstand van het lampje R.
Uit het gegeven dat het lampje zowel bij open als bij gesloten schakelaar even veel licht geeft volgt dat de stroomsterkte in beide gevallen even groot is.

Schakelaar open.

Stroom door de lamp en R_3 noemen we I_1, de stroom door R_2 noemen we I_2. De stroom door en R_1 wordt dan $I_1 + I_2$.
Dan volgt met invullen en wet van Ohm:

$$V_0 = 10(I_1 + I_2) + 20I_2 = 10I_1 + 30I_2 \qquad [1]$$

Tevens geldt:

$$20I_2 = (10 + R)I_1 \qquad [2]$$

Schakelaar dicht

Stroom door de lamp is wederom I_1, de stroom door R_1 noemen we I_3. De stroom door en R_2 wordt dan $I_1 + I_3$.
Dan volgt:

$$V_0 = 20(I_1 + I_3) + 10I_3 = 20I_1 + 30I_3 \qquad [3]$$

Tevens geldt:

$$10I_3 = I_1R \qquad [4]$$

Gelijkstellen van [1] en [3] laat V_0 wegvallen. Vervolgens [2] en [4] gebruiken om I_2 en I_3 te laten wegvallen. Dit resulteert in $R = 3{,}33\ \Omega$.